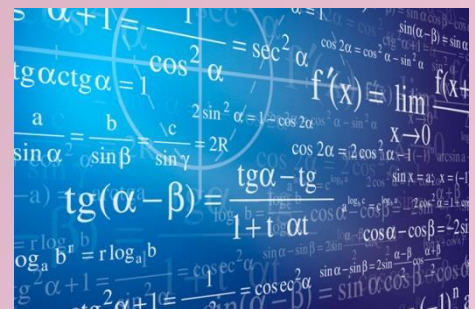
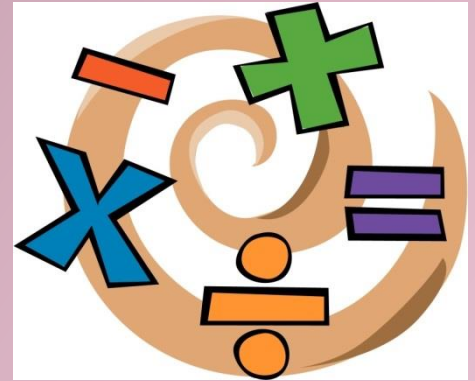


# MEM30012A

2013



Apply mathematical techniques



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Edition 1 – January 2013

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### Unit Resource Manual

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## **Aims of the Competency Unit**

This unit covers applies the concepts of mathematics to appropriate and simple engineering situations within the individual's area of engineering expertise

This unit applies to technician level work that requires basic algebraic, trigonometric and statistical knowledge and skill.

## **Unit Hours**

36 Hours

## **Prerequisites:**

None

## Elements and Performance Criteria

1. Use concepts of arithmetic in the solution of engineering problems	1.1	Units of physical quantities are converted to facilitate engineering calculations.
	1.2	Calculations are performed to solve problems involving rational and irrational numbers.
	1.3	Scientific notation is used to represent numbers.
	1.4	Calculations are checked for reasonableness using estimating and approximating techniques.
2. Solve engineering problems involving algebraic expressions with one independent variable	2.1	Algebraic expressions are manipulated using mathematical operations in their correct order.
3. Use two-dimensional geometry to solve practical problems	3.1	Angles expressed in degrees are correctly converted to radians and vice versa.
	3.2	The perimeter, area, length and angles of a range of two-dimensional figures are correctly calculated.
	3.3	The volume and surface area of complex figures are correctly calculated.
	3.4	Points identified in terms of cartesian coordinates can be converted to polar coordinates and vice versa.
4. Use trigonometry to solve practical problems	4.1	Basic trigonometry functions are used to calculate the lengths of the sides of right-angled triangles.
	4.2	Inverse trigonometry functions are used to determine angles in a right-angled triangle given the lengths of two sides.
	4.3	The sine rule is used to determine the lengths of the sides of acute and obtuse angled triangles given one side and two angles.
	4.4	The cosine rule is used to determine the lengths of the sides of acute and obtuse angled triangles given two sides and one angle.
5. Graph linear functions	5.1	Linear functions are solved graphically and equations of straight lines are determined from the slope and one point, or two points.
	5.2	Two linear functions are solved simultaneously both algebraically and geometrically.

		5.3	The length and mid-point of a line segment are determined.
6. Solve quadratic equations	quadratic	6.1	Quadratic equations are solved.
		6.2	Simultaneous linear and quadratic equations are solved.
7. Perform statistical calculations	basic	7.1	Mean, median and mode are calculated from given data.
		7.2	Standard deviation is calculated and interpreted employing graphical representation.

## Required Skills and Knowledge

### Required skills include:

- using and applying mathematical formulas
- logical thinking
- problem solving
- calculating
- applying statistics
- using computer numerical methods
- drawing graphs

### Required knowledge includes:

- transposing and evaluating formulae
- polynomials
- straight line coordinate geometry
- introduction to indices
- introduction to trigonometry
- circular functions
- trigonometry of oblique triangles
- trigonometric identities
- introduction to functions and their graphs

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## Session Program:

Session	Skill Practice Exercise
Topic 1 – Learning to Understand Formulae	MEM30012-SP-0101
Topic 2 – Using & Applying Formulae	MEM30012-SP-0201
Topic 3 – Indices & Scientific Notation	MEM30012-SP-0301
Topic 4 – Algebra	MEM30012-SP-0401
Topic 5 – Polynomials	MEM30012-SP-0501
Topic 6 – Pythagoras Theorem	MEM30012-SP-0601
Topic 7 – Trigonometry	MEM30012-SP-0701
Topic 8 – Coordinates & Statistics	MEM30012-SP-0801
Topic 9 – Straight Line Geometry	MEM30012-SP-0901
Practice Competency Test	
Answers to Skill Practice Exercises	

## Solving Maths Problems:

An equation is a combination of math expressions.

Word problems are a series of expressions that fits into an equation.

There are two steps to solving math word problems:

1. Translate the wording into a numeric equation that combines smaller "expressions".
2. Solve the equation!

### Suggestions:

- **Read the problem entirely**  
Get a feel for the whole problem
- **List information** and the variables you identify  
Attach units of measure to the variables (gallons, miles, inches, etc.)
- **Define what answer you need,**  
as well as its units of measure
- **Work in an organized manner**  
Working clearly will help you think clearly
- Draw and label all graphs and pictures clearly
- Note or explain each step of your process;  
this will help you track variables and remember their meanings
- **Look for the "key" words**  
Certain words indicate certain mathematical operations:

## Topic 1 – Learning to Understand Formulae:

### Required Skills:

- Set in place learning techniques for formulas.

### Required Knowledge:

- Understanding the advantages of studying.

### How to understand math formulas:

Many adults and students report that they find math difficult because they have trouble learning and understanding math formulae. The following points are necessary in becoming proficient in understanding math formulae:

#### ***Understanding Math is like Understanding a Foreign Language!***

Imagine a person whose native language is English and they want to read the hieroglyphics on the Great Pyramids before studying at university. All the images look very strange and they have great trouble understanding to basic message.

Before learning to read and understand a second language, say Egyptian Hieroglyphics, they would need to learn new symbols, new words and new grammar. The understandings of Egyptian scripts only comes after committing to memory hundred symbols, and have developed a reasonable understanding of Egyptian grammar.

Learning maths is no different to learning another language, new symbols (like  $\pi$ ,  $\theta$ , and  $\Sigma$ ), new words (math formulae and terms like “function” and “derivative”) and new grammar (writing equations in a logical and consistent manner). Before understanding math formulae, the meaning of the symbols, including letters, must be learnt. Concentrating on the new vocabulary is vitally important noting the way it is written and how one step follows another.

#### ***Learn the Formulas Previously Learnt from Earlier Study:***

All maths builds on maths learnt in earlier life; that is, all the new mathematical processes being learnt depend on what was learnt last week, last semester, last year and all the way back to the numbers learned as an adolescent.

If you learn formulas as you go, it will help to understand what’s going on in the new material being studied and will become better at recognizing formulas, especially when the letters or the notation are changed in small ways.

Don’t always rely on formula sheets. Commit as many formulas as possible to memory and be amazed at how much more confidence is achieved and how much better the understand each new concept.

#### ***Always Learn What the Formula and the Conditions Will Convey:***

Many students may write the quadratic formula as:

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

But this is NOT the quadratic formula! A lot of important information is missing; the parts which help to understand and apply it; all of the following is required when writing the quadratic formula. The solution for the quadratic equation is:

$$ay^2 + by + c = 0$$

For some reason, a lot of students miss out the “y =” and have no idea what the formula is doing; also, by missing out the following bit, the how and when to apply the formula is not known:

$$ay^2 + bx + c = 0$$

Learning the full situation (the complete formula and its conditions) is vital for understanding.

**Keep a chart of the formulas you need to know:**

Repetition is the key to learning; if the only time you see your math formulas is when you open your textbook, there is a good chance they will be unfamiliar and you will need to start from scratch each time. Consider a sport; a soccer player cannot bend the ball like Beckham, without practice, at golf, Tiger Woods could not sink those long putts or hit the driver long distances without long hours of practice, while in cricket, Michael Clark could not pull a high ball for six without countless hours in the nets.

Write the formulas down and write them often. Use Post-It notes or a big piece of paper and put the formulas around your bedroom, the kitchen and the bathroom. Include the conditions for each formula and a description (in words, or a graph, or a picture).

The more familiar they are, the more chance you will recognize them and the better you will understand them as they are being used.

**Math is often written in different ways - but with the same meaning:**

A lot of confusion occurs in math because of the way it is written. It often happens that one thinks one knows and understands a formula and then it will be written in another way and panic.

A simple example is the fraction “half”; it can be written as  $1/2$ , and also diagonally, as  $\frac{1}{2}$  and in a vertical arrangement like a normal fraction; it can even be expressed as a ratio, where the ratio of the 2 (equal) parts would be written 1:2.

Another example where the same concept can be written in different ways is angles, which can be written as capital letters (A), or maybe in the form  $\angle BAC$ , as Greek letters (like  $\theta$ ) or as lower case letters (y); when students become familiar with all the different ways of writing formulas and concepts, they will be able to understand them better.

Every time a new topic is started, take particular note of the way the formula is presented and the alternatives that are possible.

**Learning Formulas:**

Probably the hardest concept about mathematics is remembering the formulas. By adhering to some simple rules the process of remembering will take some of the pain out of the learning.

**Read Ahead:**

Read over next maths lesson before studying it in the learning environment. Get a general idea about the new formulas in advance, before covering them in class.

As the text read ahead, some of it will be recognised, and other parts will be brand new. That’s OK because when the teacher is explaining them, a foundation for the understanding of the new knowledge has been established and makes more sense; it will also be easier to memorize the formulas later. The technique also gives an overview of the diagrams, graphs and vocabulary in the new section where any new words can be looked up in a dictionary and reducing the stumbling blocks in class.

The step may only take 15 minutes or so, but will make a huge difference to understanding of the math you are studying.

Those who have become accustomed to reading ahead would be calmer in class while many were stressed out and confused about the new topic and its associated language.

**Use More Than One Resource:**

It often happens that students have trouble in following the teacher's explanation especially when the textbook is very confusing. Borrow 2 or 3 textbooks similar to the set text from a library and read what they have to say about the topic. Often they will have a diagram, a picture or an explanation that gives the "Ahhh – I get it!" that is wanted.

**Meaning:**

Unless blessed with a high IQ, nearly all people find it very difficult to learn meaningless lists of words, letters or numbers as our brain cannot see the connections between the arguments, and so they are quickly forgotten. Don't just try to learn formulas by themselves, it's just like learning that meaningless word and number lists.

The only formulas that need to be remembered are the common everyday formulas for area ( $L \times B$  or  $\pi r^2$ ) and volume ( $L \times B \times H$  or  $\pi r^2 H$ ) etc. If you do not know the formula, it can always be found in a book, or simply Googled. The main thing to know about formulas is HOW to use and apply them.

When the need arises to learn formulas, also learn the conditions for each formula (it might be something like "if  $x > 0$ "); another aide could be to draw a relevant diagram or graph (it might be a parabola, or perhaps a circle) each time the formula is written. After a short time the picture will be associated with the formula and then later when the formula needs to be recalled, the associated image will help in remembering it (together with its meaning, and conditions).

During exams, many students use the wrong formula to answer a question! The formula may have been successfully learnt, but have no idea of how to apply it. Most people find it difficult to learn equations in a vacuum, so make sure the formulas are learnt in their right context.

When creating a summary list of formulas, include conditions and relevant pictures, graphs and diagrams.

**Time Management:**

Start assignments as soon as they are set; there may be some things on there that have not been covered in class (because maybe it is not due for a few weeks). That's good as it helps to focus thoughts so that when the section is being covered in class, it will become apparent that it is important and any confusion or uncertainty should become clear. Nobody plans to fail – but many fail to plan...

**Don't Fall Into the Trap of Copying from a Friend to Survive:**

The problem with copying is that the other student may probably have the wrong answer, or using the formula incorrectly. Besides, a lot of students resent being asked for their assignments for copying as they are too afraid of ruining relationship by saying no. Have the confidence in your own ability – **YOU CAN DO IT.**

**Never, Never, Never Give Up:**

Maths uses a different part of the brain than most other learning; it can be stressful when something cannot be figured out; work on another subject or topic for a while and come back to it later.

**Practice:**

Addition work and skill practice exercises are not given to make students work harder or to lengthen the time of the unit; they are given as repetition is a very important aspect of learning. In repeatedly practicing a new skill, the connections between neurons in the brain are strengthened; but if the practice is not included or carried out, then the weak bonds are broken.



If trying to learn the formulas without doing the practice first, then then the learning is being made more difficult. The answer is to study, study and then study some more until a complex becomes nearly as well remembered as finding the area of a circle ( $\pi r^2$ ).

**Keep a List of Symbols:**

Most math formulas involve some Greek letters, or perhaps some strange symbols like  $\Sigma$  or perhaps  $\alpha$ .

When learning a foreign language, it's good to keep a list of the new vocabulary when discovered; as it gets more complicated, the list can be referred to, to remind us of the recently learned words but are confused about. Learning mathematical symbols should be similar too; keep a list of symbols and paste them up somewhere in the room, so they can be updated easily and can be referred to when necessary. Write out the symbol in words, for example:  $\Sigma$  is "sum";  $\int$  is the "integration" symbol and  $\pi$  is "phi", the Greek letter.

Just like when learning whole formulas, include a small diagram or graph to be reminded of where each symbol came from. Another way of keeping the list is by flash cards. Make use of dead time on the bus or train and learn a few formulas each day. Play games between classes with classmates seeing who can identify the formula the fastest.

**Use Memory Techniques:**

Most people are capable of learning lists of unrelated numbers or words, as long as they use the right techniques. Such techniques can be applied to the learning of formulas as well. One of these techniques is to create a story around the thing to be learnt. The crazier the story, the better it is because it is easier to remember. If the story is set in some striking physical location, it also helps to remember it later.

**Know Why:**

In many examinations, a math formula sheet is normally distributed so why is there a necessity to learn formulas? As mentioned earlier, if students don't know what they are doing, they will choose a formula randomly, plug in the values and hope for the best. This usually has bad outcomes and zero marks. By knowing when to use a formula will help in selecting the correct formula.

Learn the formulas, even if they are given in the exam. The process of learning the conditions for how to use the formula and the associated graphs or diagrams, means that there is a better chance of selecting and using the correct formula when answering the question. Because the student has a better grasp of the basics it helps in the future learning.

**Get on top of it before it gets on top of you.**

Mathematics is one of those subjects that builds on prior knowledge, yet many students learn only for an examination and then promptly forget it and setting themselves up for later difficulties. Learn for the future, not for test tomorrow, as those formulas in tomorrow's exam will also be used in your future engineering job.

**Sleep on it**

Don't under-estimate the importance of sleep when it comes to remembering issues. Deep sleep is a phase during the night where our brains process what was thought about during the day and this is when more permanent memories are laid down. During REM (rapid eye movement) sleep, the new skills are rehearsed and consolidated.

Avoid cramming maths formulas until late the night before an exam; have a plan for learning and spread it out so that it is not overwhelming.

**Healthy body, efficient brain**

The healthier a person is, the less is needed in worry about sickness distracting from learning. Spend time exercising and getting the oxygen flowing into the brain; this is

essential for learning. The Japanese doctrine of “healthy body, healthy mind” holds true for all cultures.

***Remove distractions***

This is a major problem for those students who are addicted to being on the Internet, twittering, texting, listening to loud music, or just talking to friends; there are just too many things that distract us from learning what we need to learn.

Turn off all those distractions for a set time each day; people won’t die without them and life will continue after ending the study session. Concentrate on the formulas needed to be learnt using all the above techniques.

When done, a reward of some music, texting or twittering time can be taken, but only after something has really been accomplished.

***Keep a Sense of Humour!***

Don’t lose the ability to laugh at yourself and your own mistakes. Mistakes are not the end of the world – they are the beginning of real learning! The best way of learning is to make mistakes – the mistake is not making mistakes or not learning from your mistakes.

**Skill Practice Exercise: MEM30012-SP-0101:**

Write a 500 word essay on how you are going to approach learning for this unit and the strategies you are going to put into place to assist in learning the formulae.